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Principles for the Prevention and Intervention of Mathematics Difficulties

Lynn S. Fuchs and Douglas Fuchs
Peabody College of Vanderbilt University

Three levels of prevention and intervention in the area of mathematics are addressed: (a) primary prevention focusing on universal design, (b) secondary prevention focusing on adaptations, and (c) tertiary prevention focusing on intensive and explicit contextualization of skills-based instruction.

Abstract. *The purpose of this paper is to identify and discuss principles of prevention and intervention in the area of mathematics. First, we identify research-based principles associated with primary prevention. Second, we turn our attention to secondary prevention, with a focus on prereferral intervention. We identify principles that serve to differentiate primary and secondary prevention and specify instructional variables that are promising for use within a secondary prevention mode. Finally, we discuss intervention. We identify principles of effective intervention, which include individually referenced decision making, instructional intensity, and deliberate contextualization of skills-based instruction.*

For students with learning disabilities (LD), mathematics problems are widespread and serious. More than 50% of students with LD have Individual Education Program goals in mathematics (Kavale & Reece, 1992), and research demonstrates the severity of mathematics difficulties for this population. For example, 6th graders with LD compute basic addition facts no better than nondisabled 3rd graders (Fleischner, Garnett, & Shepherd, 1982); less than 25% of 10th graders with LD can apply basic math knowledge (Algozzine, O'Shea, Crews, & Stoddard, 1987); the mathematics competence of students with LD progresses about 1 year for every 2 years in school (Cawley & Miller, 1989); and the math progress of students with LD eventually reaches a plateau, with little evidence of growth between the ages of 10 and 12 (Cawley, Parmar, Yan, & Miller, 1998).

In fact, prevention of mathematics difficulties in this country is generally ineffective not only for students with LD, but also for nondisabled learners. As demonstrated by Cawley et al. (1998), only 85% of normally achieving 14 year olds have mastered computational addition; 81%, subtraction; 54%, multiplication; and 54%, division. Moreover, in the International Evaluation of Educational Achievement, U.S. 8th graders performed more than 2 years behind high-scoring countries in math (Stedman, 1997).

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Unfortunately, these figures are not surprising because mathematics textbooks, which do a poor job of adhering to important instructional principles for students with and without disabilities (Jitendra, Salmento, & Haydt, 1999), account for approximately 75% of what occurs in mathematics instruction in general education (Porter, 1989). Jitendra et al. illustrated the problems associated with commercial mathematics basals when these researchers evaluated 7 mathematics texts with respect to these instructional principles: providing clear objectives, teaching 1 new concept or skill at a time, reviewing background knowledge, providing explicit explanations, structuring the use of instructional time efficiently, providing adequate practice, structuring appropriate review, and organizing effective feedback. Their analysis of the programs' treatment of subtraction across zeros, for example, revealed that only 1 series satisfied most criteria. For 2 programs, the total percentage of fully met criteria was only 33.3%.

However, a substantial body of intervention studies provides the basis for specifying methods to prevent and treat mathematics difficulties. Some frameworks

Requests for reprints should be sent to Lynn S. Fuchs, Box 328, Peabody, Vanderbilt University, Nashville, TN 37203.

conceptualize prevention at 3 levels (Forness, Kavale, MacMillan, Asarnow, & Duncan, 1996; Kauffman, 1999): primary, secondary, and tertiary.

Primary prevention focuses on universal design. With universal design, instruction for all students is formulated to incorporate principles that address the needs of specialized populations while benefiting (or at least not harming) others. An example of universal design, borrowed from everyday life, is sidewalks that dip at the curb. Although these sidewalks were designed originally to permit people in wheelchairs to cross streets, many other individuals find this sidewalk design beneficial (or at least unobtrusive). As applied to students with LD, universal design may be incorporated within general education without specialized accommodation or adaptation at the individual student level to benefit students with LD while helping other low achievers and without harming average- and high-achieving students.

Although the goal of universally designed primary prevention is to preclude the development of disorders, primary prevention does sometimes fail. In these cases, secondary prevention is offered to arrest the seriousness of the disorder or to reverse its course. Secondary prevention may be equated with prereferral intervention, whereby general education is modified in ways that are feasible for the teacher and unobtrusive for classmates. The goal is to effect better student progress with minimal invasiveness to target children and with minimal disruption to others.

By contrast, tertiary prevention is reserved for disorders that prove resistant to lower levels of prevention and require more heroic action to preclude serious complications. Tertiary prevention is synonymous with intervention, whereby intensive, individualized attention requiring special resources is brought to bear to alleviate an individual student's difficulties. The principles of tertiary intervention may apply, to varying extents, to all students. Nevertheless, it remains questionable whether these highly individualized, intensive methods are practical or even desirable for most students. Normally developing children, by definition, progress well in more naturally occurring educational environments, which also provide social benefits and are less expensive to implement. At the same time, to permit more low-achieving students to benefit from more naturally occurring educational experiences, regular programs can be strengthened with the universal instructional principles of effective instruction and with the specific primary prevention principles of mathematics instruction we discuss. Moreover, when primary prevention proves ineffectual, less intensive and less expensive prereferral intervention, or secondary prevention, may solve mathematics learning difficulties.

In this article, we adopt this 3-layered conceptualization (although we note that other frameworks exist and also work well). We structure our discussion in the following manner. First, we identify research-based principles associated with primary prevention, and we present 1 strategy for helping general educators develop a classroom routine by which they may incorporate these principles within a universal instructional design. Second, we turn our attention to secondary prevention, with a focus on prereferral intervention. We identify principles that serve to differentiate primary and secondary prevention and specify instructional variables that are promising for use within a secondary prevention mode. Finally, we discuss intervention. We identify principles of effective intervention, which include individually referenced decision making, instructional intensity, and deliberate contextualization of skills-based instruction.

PRINCIPLES OF PRIMARY PREVENTION

In discussing primary prevention principles, we focus our attention beyond the basic, well-documented instructional principles outlined by Jitendra et al. (1999). Rather, we discuss a set of instructional methods that meet 3 criteria. First, their effectiveness has been demonstrated specifically in math. Second, research illustrates the utility of these methods for students with LD. Third, these principles seem appropriate within a universal design framework because research within general education has also documented their applicability for students without LD and because they seem feasible for use within general education settings. These principles are (1) quick pace with varied instructional activities and high levels of engagement, (2) challenging standards for achievement, (3) self-verbalization methods, and (4) physical and visual representations of number concepts or problem-solving situations.

Four Principles of Mathematics Prevention

Quick Pace, Varied Activities, and Engagement

Phillips, Fuchs, Fuchs, and Hamlett (1996) identified 2 teachers for case-study analysis on the basis of students' weekly curriculum-based measurement (CBM) scores. One teacher effected the typical amount of mathematics growth among her students, which was commensurate with pupils' prior learning histories. That is, students who previously were high achieving in mathematics continued to progress well; students with poor prior learning histories in math, including 1 student with LD, continued to demonstrate minimal progress. The contrasting, more effective teacher managed to break her students' prior achievement molds. All children, including previously high-, average-, and low-achieving students, as well as those with LD, manifested high CBM growth rates in her classroom. These 2 teachers were interviewed prior to and following 1-week's worth of mathematics instruction. In addition, each of their math lessons was observed during the target week.

Several variables distinguished these teachers, including the pacing and format of their instruction. The effective teacher incorporated a dramatically quicker pace, and this faster pace resulted in more activities in every lesson. Moreover, as the effective teacher incorporated more instructional activities, she also relied on a greater range of grouping arrangements. As might be expected, the effective teacher's quick instructional pacing and varied instructional formats led to more active student involvement. Her students participated by discussing, writing, computing, and problem solving almost 100% of the time. This contrasted sharply with the level of student engagement effected by the more typical teacher, where students' primary responsibility was to sit and listen.

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These findings, based on an in-depth analysis of 2 teachers in the area of mathematics instruction, corroborate research on effective teaching in general. Brophy and Alleman (1991), for example, found that smooth instructional pacing is essential for implementing effective instructional activities and promoting student learning. Moreover, studies suggest that instruction should include a variety of formats and student response modes because such variety accommodates individual differences (Brophy & Alleman, 1991; Fraenkel, 1980).

Challenging Achievement Standards

Another key variable distinguishing these 2 teachers concerned the level and type of motivation they provided their students. The effective teacher simply devoted more effort to motivating her students: she incorporated 6 times more motivating statements and activities into her lessons. Beyond simple quantity, however, the style these 2 teachers used in attempts to motivate students also differed. The less effective teacher's motivational statements were designed to convince students that upcoming activities would be fun and interesting. By contrast, the more effective teacher's motivators were constructed to communicate high expectations: she expected that everyone in the class would learn.

With these uniformly high expectations, all children in the class appeared eager to learn and seemed highly engaged in the mathematics activities. Moreover, their CBM scores during the period we conducted the study indicated that the motivational environment promoted high levels of learning among all students, including those with LD. And, interestingly, the high expectations of the effective teacher mirror current reform efforts to promote challenging standards for all students (McDonnell, McLaughlin, & Morison, 1997).

Self-Verbalization

Research in mathematics has specifically identified cognitive strategy instruction as an effective instructional tool. Students are taught and memorize explicit steps for approaching and solving problems, and they apply these steps by verbalizing them, first overtly and gradually fading their overt use over time. For instance, Montague, Applegate, and Marquard (1993) used cognitive strategy instruction to enhance problem-solving performance. Students with LD were taught (1) 7 cognitive steps (read the problem, paraphrase, visualize with a picture or diagram, hypothesize a plan to solve the problem, estimate the answer, compute, and check) or (2) methods for regulating their use of these steps, or (3) both. Results indicated that the 7 cognitive steps were sufficient to effect change in the first 7 days. However, with additional time, students in all 3 treatments showed meaningful improvement.

In a related way, Hutchinson (1993) provided students with self-questions on cue cards, structured worksheets, teacher modeling of the strategy, and prompts, along with corrective feedback and reinforcement. Twenty students with LD were randomly assigned either to this treatment or to typical classroom instruction. Students in the experimental group outperformed control group counterparts on 3 types of algebraic word problems, and a majority of students maintained results for 6 weeks. Leone and Pepe (1983) also demonstrated how self-verbalization methods enhance mathematics learning for students with LD.

These studies are representative of a literature base (see Maccini & Hughes, 1997; Mastropieri et al., 1991; Xin & Jitendra, 1999, for reviews) providing evidence that verbal rehearsal routines, which specify steps for approaching and solving mathematics problems, are an effective means for enhancing the performance of students with LD. These methods represent an important component for prevention of mathematics difficulties for students with LD. Moreover, within a universal design framework, it is important to note that mathematics instruction incorporating self-verbalization strategies also has been shown to be effective for low-, average-, and high-performing students without LD (Fuchs, Fuchs, Hamlett, Phillips, & Karns, l995).

Physical and Visual Representations

For decades, research on mathematics instruction has focused on procedures more than concepts (see Cawley et al., 1998). Yet, studies demonstrate the importance of conceptual understanding, not only to facilitate application of procedural knowledge, but also to accomplish long-term retention of procedural competence. For example, Woodward, Howard, and Battle (1997) demonstrated that low-performing 3rd graders without disabilities who were taught subtraction to mastery using only procedural methods tended to regress significantly in their performance once the teacher moved to another math topic.

However, research demonstrates that using physical and visual representations to facilitate conceptual understanding helps children master and maintain mathematical competence. For example, working with 12 students with LD along with their nondisabled classmates in 6 classrooms, Harris, Miller, and Mercer (1995) taught multiplication skills using a concrete-to-representational-to-abstract instructional sequence. Importantly, this carefully designed treatment, which began with physical and visual representations of multiplication, was implemented within the context of carefully structured instruction that incorporated goals, direct instruction, specific questions, guided and independent practice, student achievement monitoring, behavior management, and generalization programming. Results showed that students with and without LD learned multiplication to high levels of competence, which matched their normally achieving peers in all phases of instruction except word problems. This and related work (e.g., Kelley, Gersten, & Carnine, 1990; Mercer & Miller, 1992; Peterson, Mercer, & O'Shea, 1988; Woodward, Baxter, & Robinson, 1999) suggest the importance of visual and physical representations within prevention programs.

Helping General Education Integrate These Principles

To help general educators integrate principles for effective mathematics instruction with students with and without LD in order to achieve universal design, we developed Peer-Assisted Learning Strategies, or PALS, for use in general education. PALS incorporates basic instructional principles (Jitendra et al., 1999) such as monitoring student achievement, clear objectives, explicit instruction, specific questions, guided and independent practice, and elaborated feedback. It also relies on a strong motivational system; quick pace, varied activities, and high levels of engagement; self-verbalization methods; and physical and visual representations of number concepts or problem-solving situations. Moreover, and importantly for a preventive approach, PALS is feasible for teachers to use and is unobtrusive for and beneficial to all types of students, including those with and without learning difficulties. Below, we provide an overview of PALS and clarify how it addresses primary prevention principles.

As part of PALS, teachers employ weekly CBM to track pupil progress toward proficiency on the grade-level mathematics operations and applications curricula. Using a standard-measurement task, teachers assess each pupil's performance weekly, each time on a different assessment representing the grade-level's annual curriculum. Each CBM assessment comprises approximately 45 problems; the exact number depends on grade level and remains constant within grade level. Teachers administer the weekly assessments in a whole-class format, using an audiotape to signal the beginning and end of the test. Students enter their responses into a computer program that scores and manages the data.

This software summarizes each pupil's performance in terms of (1) a graph displaying the total number of correct problems over time and (2) a skills profile showing the student's mastery status on each type of problem included in the year's curriculum for each half-month interval in the school year. Teachers instruct students on how to read and interpret graphs and skills profiles and teach students to ask themselves 3 questions about their graphs (Are my scores going up? What's my highest score? Can I beat my highest score in the next 2 weeks?) and about their skills profiles (Are my boxes getting darker? How many black or almost-black boxes do I have for this half-month? Which skills can I work harder on in order to get darker boxes the next half month?). Every 2 weeks, when graph and skills profile feedback is provided to students, teachers remind students to ask themselves these questions.

Twice monthly, teachers print a copy of each student's graph and skills profile, as well as a teacher report summarizing the performance of the class. This report includes the following descriptive information: (1) a class graph displaying students' total number correct over time at the 25th, 50th, and 75th percentiles of the class; (2) a list of pupils whose current performance falls below the 25th percentile; (3) lists of skills on which student performance has improved, stayed the same, and deteriorated over the past month; and (4) a class skills profile displaying every student's mastery status for the current half-month interval on each problem type in the year's curriculum and providing a frequency count of the numbers of students in each mastery status for each problem type. The report also includes instructional recommendations for (1) what to teach during whole-class instruction; (2) how to constitute small groups of students for instruction on skills with which students experience common chronic difficulty; (3) computer-assisted instruction, listing the skill and the computer-assisted program each student should use for the next 2 weeks; and (4) peer-assisted learning strategies, listing students who require and those who can provide assistance with which skills.

Computer-managed CBM provides a routine, feasible mechanism for monitoring student progress and for structuring a system by which each student can develop a strong goal orientation and teachers can motivate students to focus on and improve learning. At the same time, the teacher feedback permits easy identification of appropriate skills to target for instruction and clear specification of those objectives.

In addition, with PALS, teachers incorporate 2 35-minute peer-assisted learning sessions each week into their allocated mathematics time. Teachers use PALS to help remediate or to review portions of the curriculum already addressed during teacher-directed instruction. Teachers teach the PALS routine to children in 5 30-minute sessions (see Fuchs, Fuchs, Karns, & Phillips, 1999, for a manual providing scripted lessons in both CBM and PALS).

PALS borrows its basic structure from ClassWide Peer Tutoring (Greenwood, Delquadri, & Hall, 1989), in which every child in the class is paired to work with another child in the same class. PALS extends ClassWide Peer Tutoring by employing a dyadic structure based on the following design features: (1) mediated verbal rehearsal, in which the tutor models and gradually fades a verbal rehearsal routine delineating procedural steps for completing the problem type; (2) step-by-step feedback by the tutor to confirm and praise correct responses and to provide explicit explanations and model strategic behavior for incorrect answers; (3) frequent verbal and written interaction between tutors and tutees; (4) opportunities for tutees to apply explanations in subsequent problems; and (5) reciprocity, where both children serve in the roles of tutor and tutee within each session. We incorporated these design features into PALS based on research documenting the potential for mediated verbal rehearsal (Graham & Harris, 1989; Zook & DiVesta, 1989), appropriate feedback for learner responses (Walberg, 1984), opportunity for learner responding (e.g., Greenwood et al., 1989) and for constructive activity following explanations (Webb, Troper, & Fall, l995), and reciprocity (Top & Osguthorpe, 1987; Wiegman, Dansereau, & Patterson, 1992) to enhance learning outcomes. PALS relies on a structured interaction because research (Fitz-Gibbon, 1977; Michaels & Bruce, 1991; Palincsar & Brown, 1989) indicates that open-ended discussions and explanations frequently are problematic, confused, and ineffective.

During PALS, every student in the class is paired to work on a skill with which one student requires assistance and the other child can provide help. PALS activities are designed for the comprehensive mathematics curriculum, so that different dyads can simultaneously work on number concepts, counting, word problems, charts/graphs, money, measurement, geometry, or computation. During PALS, each dyad works through 12–20 instances of its target problem type on a problem sheet. The tutor models a series of verbal statements or questions that the tutee can use as a guide to the problem's solution. Each statement requires a verbal or written action by the tutee. Statements differ by problem type. Tutors respond every time the tutee writes any response. When the tutee is correct, the tutor circles the response and praises the tutee; when the tutee is incorrect or expresses confusion, the tutor provides as much additional help as is necessary. Consequently, the nature of this additional help is not structured and requires tutors to construct their own explanations routinely.

The problem sheet is divided into 4 problem sets of equal length. With the first set, the dyad completes the explanatory interaction just described. The tutee works the next problem set more independently, explaining work back to the tutor while the tutor listens, corrects incorrect statements, and relies on the correction procedure used with the first problem set. Then, the 2 students reverse roles and repeat the same sequence of activities. Thus, PALS involves gradual fading of a verbal rehearsal routine that incorporates high levels of feedback and participation by both students, with children sharing the roles of teacher and student and routinely constructing explanations. Every 2 weeks, tutoring assignments change, and are based on weekly CBMs.

Two weeks after PALS begins, 2 types of helping and explaining lessons are introduced to enhance the quality of peer interactions and explanations. The first lesson covers principles for seeking elaborated help (i.e., ask for help; keep on asking until you understand) and for offering elaborated help (i.e., pay careful attention to your partner; if you think your partner needs help, offer to help; don't just give the answer, explain how your partner can find the answer; if one explanation does not help, try another; ask your partner to explain your explanation back to you to find out if he or she really understands).

The second set of 3 lessons, each of which relies on videotaped vignettes, covers methods for providing conceptual mathematical explanations. These 3 lessons, lasting 40, 40, and 15 minutes, encourage students to contextualize problem situations, to represent quantities with visual images or physical materials, and to discuss solution strategies. PALS translates these goals into 5 methods students can use to provide conceptual mathematical explanations to peers: (1) build number sentences incorporating real-life examples that are interesting and easy to picture in your head; (2) make marks or pictures that stand for the numbers; (3) use manipulatives so your partner can move and touch things that stand for the numbers; (4) discuss the meaning of the numbers by explaining what the numbers stand for, talking about *why* the problem must be worked in a certain way, or discussing if and why the answer does or does not make sense; and (5) ask step-by-step questions that begin with *what*, *where*, *when*, *how*, and *why* (see Fuchs, Fuchs, Hamlett et al., 1997, for examples of each method). Once each week, following a PALS session, teachers lead a 5-minute debriefing session in which they (1) ask if anyone has received an explanation that really helped; (2) ask children to describe how they decided what their tutees needed help with; (3) solicit descriptions of helpful explanations; (4) require the class to classify the explanations; and (5) refer at least once to each of the 5 methods.

The combined CBM and PALS methods have been shown to effect better mathematics achievement among a range of students, including those with LD, who participate in general education classrooms (e.g., Fuchs, Fuchs, Hamlett et al., 1997) and have been designated an "effective practice" by the Program Effectiveness Panel in the U.S. Department of Education. The combined method incorporates the principles of effective teaching, generally, and effective mathematics prevention, specifically, into a "routine" that teachers can easily manage and use. In a similar way, we have also developed and validated PALS mathematics programs at the kindergarten (Fuchs, Fuchs, & Karns, in press) and 1st-grade level (Fuchs, Fuchs, Karns, & Ardman, 2000), which serve as useful frameworks to help teachers routinize teaching practices that promote the prevention of mathematics difficulties.

PRINCIPLES OF SECONDARY PREVENTION: PREREFERRAL INTERVENTION

Even with successful prevention programs, some children do not respond. With PALS, for example, we find that approximately 15% of students fail to make better progress than would be expected in typical general education classrooms and, according to CBM normative frameworks, fail to manifest acceptable levels of growth. Consequently, even with strong prevention programs, a mechanism to address student difficulties will be necessary. The next mechanism is secondary prevention, which is known within special education as prereferral intervention.

Even with successful prevention programs, some children do not respond.

With secondary prevention, or prereferral intervention, the general education setting is modified with 3 principles in mind. The first principle is that adaptations must be feasible for the general educator to implement within the normal classroom routine. The second principle is that the adaptations cannot be disruptive to the target child. The third principle is that the adaptations must be nonintrusive for classmates. The goal is to effect better student responsiveness (1) without heroic action on behalf of the general educator, who is responsible for a large number of children, (2) with minimal invasiveness to the target child, and (3) with minimal disruption to others. Of course, secondary prevention offers the classroom teacher some additional structure via consultation with experts (e.g., the special educator or the school psychologist) or via collaboration with fellow teachers (e.g., student-support teams). Consultation or collaboration allows the infusion of fresh, potentially effective strategies to address the needs of the target child within the framework of the general education system. Below, we provide an example of secondary prevention with a study we conducted relying on the PALS structure just explained.

Fuchs et al. (1995) randomly assigned 20 general educators to 2 treatments. PALS represented the baseline treatment; that is, in both treatments, teachers implemented math PALS with all students in their classes beginning in September. This, along with other components of the teachers' mathematics instruction, represented the primary prevention program.

In light of research showing that primary prevention programs, including PALS, are not effective for every individual, our contrast treatment systematically incorporated secondary prevention. This secondary prevention focused on specialized adaptations, conducted within regular classrooms, for the subset of students who manifested unacceptable performance and growth. Beginning in November, the bimonthly CBM class reports identified up to 2 target students whose CBM progress was inadequate (i.e., low level, in combination with low slope of improvement over time, relative to classmates). For these students, teachers (1) formulated an adaptation before the next 2-week report; (2) implemented that adaptation at least 4 times in the upcoming 2 weeks; and (3) when CBM identified the same student multiple times over reports, modified previous adaptations to enhance progress.

To assist teachers, we structured a brief diagnostic that focused on qualitative dimensions of the CBM data as well as the target student's classroom performance. The teacher used this diagnostic to classify the source of the student's problem as poor motivation, disfluency with basic facts, lack of conceptual understanding, careless work habits, or other. For the first 4 categories, we developed a taxonomy of adaptations, deemed feasible for general education implementation, and provided "adaptation kits" for several activities within the taxonomy.

Across 3 to 6 2-week adaptation cycles, teachers ignored 108 requests for adaptations only 4 times; they implemented multiple strategies concurrently to address the problems of target students 17 times; and some teachers manifested an impressive level of dedication by modifying student programs repeatedly in a variety of ways in an attempt to boost progress. Teachers' reliance on individual adaptations also appeared to prompt changes in their thinking about their role in and use of secondary prevention. Compared to teachers in the baseline treatment, those in the specialized adaptations treatment reported (1) more modifications in their goals and strategies for poorly progressing students; (2) a greater variety of skills taught; (3) more frequent reteaching of selective lessons; and (4) more frequent deviation from the teacher's manual for selected students. Moreover, for some of these students, most of whom were students with LD, meaningful improvements in students' progress were effected.

This suggests that general educators, with consultative or collaborative support, can make adaptations to students' mathematics programs that (1) are feasible for implementation within the natural classroom environment; (2) are not disruptive to the target child; (3) are not intrusive for classmates; and (4) can be effective in addressing individual student difficulties. In this effort, our "adaptation kits," or secondary prevention strategies, were goal setting (e.g., Fuchs, Bahr, & Rieth, 1989; Schunk, 1985), self-monitoring of task completion and work quality (e.g., Bahr, Fuchs, Fuchs, Fernstrom, & Stecker, 1993), computer-assisted instruction (e.g., Bahr & Rieth, 1991; Gleason, Carnine, & Boriero, 1990; Howell, Sidorenko, & Jurica, 1987; Shiah, Mastropieri, Scruggs, & Fulk, 1995; Trifiletti, Frith, & Armstrong, 1984), and concrete representations of numbers and number concepts (e.g., Kelley et al., 1990; Mercer & Miller, 1992; Peterson et al., 1988; Woodward et al., 1999). Another research-based instructional method that seems potentially productive at the secondary prevention level is reinforcement (Albert & Greer, 1991; McLaughlin & Helm, 1993; Smith & Lovitt, 1976).

Of course, it is important to note that within this, or any, secondary prevention effort, findings were not uniformly positive. Most important, despite many focused, data-based attempts to enhance learning, some children proved unresponsive to regular classroom adaptations. Two brief cases illustrate students' differential responsiveness.

Over a 12-week period, a 4th-grade teacher implemented many diverse adaptations, relying on basic facts drill, motivational workcharts and contracts, and manipulatives. The target student, who had exhibited a poor CBM rate of improvement (i.e., 0.21 digits per week) when identified for adaptation, responded well to these modifications in the general education classroom and completed the school year with a CBM rate of improvement of 0.63 digits per week, the average slope for the class.

This success contrasts with the experience of a 3rd-grade teacher who also implemented a large number and rich set of adaptations, including drilling basic facts, slicing back to 2nd-grade material, implementing a motivational workchart, and using money to work on conceptual underpinnings. Despite this teacher's level of effort to modify regular classroom instruction, her target student demonstrated little improvement in growth rate: he ended the year with a relatively low CBM rate of improvement (i.e., 0.28 digits per week), which was similar to his CBM rate of growth at the time he was identified for specialized adaptations. By contrast, his classmates' average CBM rate of improvement was 0.98 digits per week.

Three of our 10 teachers effected substantial improvement for their target students. This suggests that with (1) the assistance of rich assessment information, (2) a routine structure within which to incorporate adaptations, and (3) consultative support to formulate feasible adaptations, regular classroom teachers can address the problems of some portion (in this case, 30%) of their students who initially demonstrate significant learning discrepancies.

Unfortunately, it is important to note that this database simultaneously indicates that some students will remain unresponsive to an adapted general education environment. This unresponsiveness to secondary prevention creates the need for additional resources, specifically for the individualized instruction, the small-size instructional groups, and the more highly trained teachers available through special education. This constitutes the third level of prevention: intervention.

PRINCIPLES OF INTERVENTION

Tertiary prevention is synonymous with intervention. With intervention, intensive and individualized attention, which requires special resources, is brought to bear on problems that are severe and have proved resistant to other levels of prevention. Typically, special educators provide this level of service. Three instructional principles distinguish intervention from primary and secondary prevention. Each principle is supported by research as important to enhance learning among students with LD generally and specifically in mathematics: (1) a focus on the individual student as the unit for instructional decision making, (2) intensive instructional delivery, and (3) explicit contextualization of skills-based instruction.

Individually Referenced Decision Making

Tertiary prevention, or intervention, centers instructional decision making on the individual student. Individually focused instructional decision making specifies research-based methods for tracking student progress and for using the resulting database to formulate ambitious learning goals and to test alternative hypotheses about the instructional methods that produce satisfactory growth. Over time, the teacher empirically tests and develops an instructional program tailored for the individual student (see Fuchs & Fuchs, 1998, for review).

Individually referenced decision making is perhaps the signature feature of effective special education intervention. It fosters high expectations for learning. It requires teachers to reserve judgment about the efficacy of an instructional method for a student until the method either does or does not prove effective for that individual. It necessitates a form of teacher planning that incorporates ongoing, major adjustments and revisions in response to an individual student's learning. And it requires knowledge of multiple ways to adapt curricula, modify instructional methods, and motivate students.

Evidence documents how individually referenced decision making enhances learning for students with LD. A meta-analysis summarized the efficacy of individually referenced decision making for students with disabilities with an effect size of 0.70 standard deviations (Fuchs & Fuchs, 1986); more recent studies in mathematics (e.g., Fuchs, Fuchs, Hamlett, & Stecker, 1990, 1991) corroborate earlier effect sizes. Stecker and Fuchs (2000), for example, assessed the added value of individually referenced decision making over and beyond the effects of regularly introducing instructional revisions (i.e., simply varying the instructional program at regular intervals) and of routinely measuring student performance (i.e., simply conducting CBM). Pairs of students with disabilities were matched. The performance of one randomly selected student in each pair was measured twice weekly, and the teacher formulated instructional decisions for both students in the pair based on this one student's assessment profile. Moreover, half the matched students were also measured, but teachers had no access to their assessment profiles. Results showed that students whose instructional decisions were tailored to their own ongoing assessment profiles achieved reliably better than their matched pairs, and that measurement alone contributed little to student achievement.

Intensive Instruction

Meta-analyses and narrative syntheses (Cohen, Kulik, & Kulik, 1982; Glass, Cahen, Smith, & Filby, 1982) show that intensive instruction can result in impressive learning for students who otherwise fail to achieve critical benchmarks using primary and secondary levels of prevention (Glass, McGaw, & Smith, 1981).

Fuchs, Fuchs, and Fernstrom (1993) illustrated the importance of this principle. These researchers worked with special education teachers as they planned to reintegrate 21 students into mathematics instruction with general education classrooms. These special educators formulated instructional decisions on the basis of each student's individual CBM data and worked intensively with each student, providing carefully structured one-on-one instruction or small-group instruction. The goal was to effect mathematics competence commensurate with the lowest acceptable performance level of students in the classroom into which the target student would reintegrate. To determine this level, special educators collected CBM data not only on the target student but also on 3 low-performing (but legitimate academic) members of the general education setting. When the special educator succeeded in effecting sufficient math growth so that the target student's performance level approached that of the low-performing peers, reintegration occurred. At that time, the onus for instruction transferred to the regular classroom teacher, and intervention switched into a prevention mode.

Results are interesting for 2 reasons. First, the study illustrates how intensive instruction can produce excellent growth rates among students with LD. The experimental students' growth rates with the special education treatment far exceeded those of a comparison group of students with LD for whom special education failed to incorporate intensive instruction. In addition, the experimental students' rates of growth far exceeded that of the low-performing students within general education. The second reason the results of this study are instructive is the pattern of performance following reintegration. After the transition to regular classrooms, CBM data continued to be collected for the target student and for the low-performing peers in the regular classroom. During the special education period, the experimental students' slopes were significantly greater than those of the low-performing peers. However, after reintegration, the slopes of the target students plunged and were significantly lower than those of the comparison students. On average, 63% of the reintegrated students' CBM data points in regular education fell below the trend lines that had been projected on the basis of their growth rates with intensive special education. This illustrates how intensive instruction can be critical for many students with LD.

It is important to note that although one-on-one tutoring may be necessary to achieve instructional intensity and promote learning, intensive instruction is not synonymous with one-on-one delivery. Rather, intensive instruction refers to a broader set of instructional features including, but not limited to, (1) high rates of active responding at appropriate levels, (2) careful matching of instruction with the individual student's skill levels, (3) instructional cues, prompts, and fading to support approximations to correct responding, and (4) detailed task-focused feedback—all features that may be incorporated into group lessons (see, for example, the work of Mark Wolery et al.).

Explicitly Contextualizing Skills-Based Instruction

Despite the questionable pertinence of constructivist assumptions when designing programs for students with LD, it is important to note that constructivist philosophy has influenced current conceptualizations of effective tertiary prevention, or special education intervention practice, in substantial ways. The notion of isolated skills instruction has been replaced with more contextualized presentations, where strategies for applying skills within generalized contexts are taught explicitly. Research documents the potential value of situating explicit skills instruction within structured, motivating, and authentic contextualized applications for knowledge application to occur.

For example, Fuchs, Fuchs, Prentice, Burch, Hamlett, Owens, Hosp, and Jancek (2000) experimented with methods to promote transfer of math problem-solving skills to novel contexts. Over 4 months, the following problem-solving skills were taught: finding information to solve problems, organizing work, buying things in sets, using half, and using pictographs. A conventional teach-and-drill condition was contrasted with a meta-level treatment. This meta-level condition situated skills instruction within contextualized learning experiences. Students were taught what the transfer of skills means in mathematics; they were taught 4 ways the transfer occurs in mathematics (i.e., problems can look different, can ask questions in different ways, can use different vocabulary, and can imbed skills within larger problem-solving contexts); and they had routine opportunities to apply skills under teacher direction. We found that 3rd graders in meta-level treatments, including those with LD, displayed greater problem-solving capacity within novel formats compared to peers in the teach-and-drill treatment.

Consequently, for students who fail to respond to prevention efforts, data-based arguments support a situated approach to teaching that blends explicit teaching of skills with contextually rich learning experiences; this position echoes important principles of constructivism. Nevertheless, it is clear that explicit teaching is fundamental even within this situated teaching approach: the teacher reveals or makes transparent the connections between knowledge acquisition and knowledge application, rather than leaving the student to discover those connections more incidentally.

SUMMARY

Together, individually referenced decision making, intensive instruction, and explicit contextualization of skills-based instruction represent a potent set of instructional practices that have been demonstrated to promote learning for students with LD. These 3 principles constitute tertiary prevention, or intervention. The research base on the specific interventions subsumed under these broad principles documents large effect sizes ranging from 0.50 to over 1.50 standard deviations.

These 3 instructional features are not only potentially effective; they also represent principles strikingly different from those of primary and secondary levels of prevention. Specifically, intervention focuses on the individual as the unit of analysis, whereas primary prevention relies on the group. Although secondary prevention does locate instructional decisions with individual students, it incorporates a limited set of instructional methods that preserve feasibility for implementation by the general educator and nonintrusiveness for target students and classmates. Universally designed, nonintensive primary prevention principles should meet the needs of nondisabled students in the mainstream, as well as some portion of students with LD. Tertiary prevention, by contrast, requires intensive instruction and careful, explicit contextualization of skills-based instruction. Tertiary prevention, therefore, is reserved for the subset of students for whom primary as well as secondary prevention proves unsuccessful.

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REFERENCES

Albert, A. E., & Greer, R. D. (1991). Is the three-term contingency trial a predictor of effective instruction? *Journal of Behavioral Education, 1,* 331–354.

Algozzine, B., O'Shea, D. J., Crews, W. B., & Stoddard, K. (1987). Analysis of mathematics competence of learning disabled adolescents. *Journal of Special Education, 21,* 97–107.

Bahr, C. M., & Rieth, H. J. (1991). The effects of instructional computer games and drill and practice software on learning disabled students' mathematics achievement. *Computers in the Schools, 6*(3/4), 87–101.

Bahr, M. W., Fuchs, D., Fuchs, L. S., Fernstrom, P., & Stecker, P. (1993). Effectiveness of student versus teacher monitoring during prereferral intervention. *Exceptionality, 4,* 48–54.

Brophy, J., & Alleman, J. (1991). Activities as instructional tools: A framework for analysis and evaluation. *Educational Researcher, 20*(4), 9–23.

Cawley, J. F., & Miller, J. H. (1989). Cross-sectional comparisons of the mathematics performance of children with learning disabilities: Are we on the right track toward comprehensive programming? *Journal of Learning Disabilities, 22,* 250–254.

Cawley, J. F., Parmar, R. S., Yan, W., & Miller, J. H. (1998). Arithmetic computation performance of students with learning disabilities: Implications for curriculum. *Learning Disabilities Research & Practice, 13,* 68–74.

Cohen, P., Kulik, J. A., & Kulik, C. (1982). Educational outcomes of tutoring: A meta-analysis of findings. *American Educational Research Journal, 19,* 237–248.

Fitz-Gibbon, C. T. (1977). An analysis of the literature of cross-age tutoring. ERIC Document Reproduction Service No. ED 148-807.

Fleischner, J. E., Garnett, K., & Shepherd, M. J. (1982). Proficiency in basic fact computation of learning disabled and nondisabled children. *Focus on Learning Problems in Mathematics, 4,* 47–55.

Forness, S. R., Kavale, K. A., MacMillan, D. L., Asarnow, J. R., & Duncan, B. B. (1996). Early detection and prevention of emotional or behavioral disorders: Developmental aspects of systems of care. *Behavioral Disorders, 21,* 226–240.

Fraenkel, J. (1980). *Helping students think and value: Strategies for teaching the social studies* (2nd ed.). Englewood Cliffs, NJ: Prentice-Hall.

Fuchs, D., Fuchs, L. S., & Fernstrom, P. J. (1993). A conservative approach to special education reform: Mainstreaming through transenvironmental programming and curriculum-based measurement. *American Educational Research Journal, 30,* 39–46.

Fuchs, L. S., Bahr, C. M., & Rieth, H. J. (1989). Effects of goal structures and performance contingencies on math performance of adolescents with learning disabilities. *Journal of Learning Disabilities, 22,* 554–560.

Fuchs, L. S., & Fuchs, D. (1986). Effects of systematic formative evaluation on student achievement: A meta-analysis. *Exceptional Children, 53,* 199–208.

Fuchs, L. S., & Fuchs, D. (1998). Treatment validity: A unifying concept for reconceptualizing the identification of learning disabilities. *Learning Disabilities Research & Practice, 13,* 204–219.

Fuchs, L. S., Fuchs, D., Hamlett, C. L., Phillips, N., & Karns, K. (1995). General educators' specialized adaptation for students with learning disabilities. *Exceptional Children, 61,* 440–459.

Fuchs, L. S., Fuchs, D., Hamlett, C. L., Phillips, N., Karns, K., & Dutka, S. (1997). Enhancing students' helping behavior during peer-mediated instruction with conceptual mathematics explanations. *Elementary School Journal, 97,* 223–250.

Fuchs, L. S., Fuchs, D., Hamlett, C. L., Stecker, P. M. (1990). The role of skills analysis in curriculum-based measurement in math school. *Psychology Review, 19*, 6–22.

Fuchs, L. S., Fuchs, D., Karns, K., Hamlett, C. L., Katzaroff, M., & Dutka, S. (1997). Effects of task-focused goals on low achievers with and without learning disabilities. *American Educational Research Journal, 34,* 513–544.

Fuchs, L. S., Fuchs, D., & Karns, K. (in press). Enhancing kindergarten children's mathematics development: Effects of peer-assisted learning strategies. *Elementary School Journal*.

Fuchs, L. S., Fuchs, D., Karns, K., & Ardman, L. (2000). *Effects of peer-assisted learning strategies on the development of mathematics competence among first-grade students with and without disabilities*. Manuscript in preparation.

Fuchs, L. S., Fuchs, D., Karns, K., & Phillips, N. B. (1999). *Peer-Assisted Learning Strategies in math: A manual*. Available from author.

Fuchs, L. S., Fuchs, D., Hamlett, C. L., & Stecker, P. M. (1991). Effects of curriculum-based measurement and consultation on teacher planning and student achievement in mathematics operations. *American Educational Research Journal, 28,* 617–641.

Fuchs, L. S., Fuchs, D., Prentice, K., Burch, M., Hamlett, C. L., Owens, R., Hosp, M., & Jancek, D. (2000). *Explicitly teaching for transfer: Effects on third-grade students' mathematical problem solving*. Manuscript in preparation.

Glass, G., Cahen, L., Smith, M. L., & Filby, N. (1982). *School class size*. Beverly Hills, CA: Sage.

Glass, G., McGaw, B., & Smith, M. L. (1981). *Meta-analysis in social research*. Beverly Hills, CA: Sage.

Gleason, M., Carnine, D., & Boriero, D. (1990). Improving CAI effectiveness with attention to instructional design in teaching story problems to mildly handicapped students. *Journal of Special Education Technology, 10,* 129–135.

Grahom, S. E., Harris, K. R. (1989). Components analysis of cognitive strategy instruction: Effects on learning disabled students' compositions and self efficiacy. *Journal of Educational Psychology, 81*, 353–361.

Greenwood, C. R., Delquadri, J. C., & Hall, R. V. (1989). Longitudinal effects of classwide peer tutoring. *Journal of Educational Psychology, 81,* 371–383.

Harris, C. A., Miller, S. P., & Mercer, C. D. (1995). Teaching initial multiplication skills to students with disabilities in general education classrooms. *Learning Disabilities Research & Practice, 10,* 190–195.

Howell, R., Sidorenko, E., & Jurica, J. (1987). The effects of computer use on the acquisition of multiplication facts by students with learning disabilities. *Journal of Learning Disabilities, 20,* 336–341.

Hutchinson, N. L. (1993). Effects of cognitive strategy instruction on algebra problem solving of adolescents with learning disabilities. *Learning Disability Quarterly, 16,* 34–63.

Jitendra, A. K., Salmento, M. M., & Haydt, L. A. (1999). A case analysis of fourth-grade subtraction instruction in basal mathematics programs: Adherence to important instructional design criteria. *Learning Disabilities Research & Practice, 14,* 69–79.

Juel, D. (1996). What makes literacy tutoring effective? *Reading Research Quarterly, 31,* 268–289.

Kavale, K. A., & Reese, J. H. (1992). The character of learning disabilities. *Learning Disability Quarterly, 15,* 74–94.

Kelly, B., Gersten, R., & Carnine, D. (1990). Student error patterns as a function of curriculum design: Teaching fractions to remedial high school students and high school students with learning disabilities. *Journal of Learning Disabilities, 23,* 23–29.

Leone, J. A., & Pepe, H. J. (1983). Self-instructional training: Cognitive behavior modification for remediating arithmetic deficits. *Exceptional Children, 50,* 54–60.

Locke, E. A., Shaw, K. N., Saari, L. M., & Latham, G. P. (1981). Goal-setting and task performance: 1969–1980. *Psychological Bulletin, 90,* 125–152.

Maccini, P., & Hughes, C. A. (1997). Mathematics interventions for adolescents with learning disabilities. *Learning Disabilities Research & Practice, 12,* 168–176.

Mastropieri, M. A., Scruggs, T. E., & Shiah, S. (1991). Mathematics instruction for learning disabled students: A review of research. *Learning Disabilities Research & Practice, 6,* 89–98.

McLaughlin, T. F., & Helm, J. L. (1993). Use of contingent music to increase academic performance of middle school students. *Psychological Reports, 72,* 658.

Mercer, C. D., & Miller, S. P. (1992). Teaching students with learning problems to acquire, understand, and apply basic math facts. *Remedial and Special Education, 13*(3), 19–35.

Michaels, S., & Bruce, C. (1991). Discourses on the seasons. Technical Report. Champaign, IL: University of Illinois, Reading Research and Education Center.

Montague, M., Applegate, B., & Marquard, K. (1993). Cognitive strategy instruction and mathematical problem-solving performance of students with learning disabilities. *Learning Disabilities Research & Practice, 8,* 223–232.

Palincsar, A. S., & Brown, A. L. (1989). Classroom dialogues to promote self-regulated comprehension. In J. Brophy (Ed.), *Advances in research on teaching* (vol. 1, pp. 35–71). New York: JAI.

Peterson, S. K., Mercer, C. D., & O'Shea, L. (1988). Teaching learning disabled students place value using the concrete to abstract sequence. *Learning Disabilities Research, 4,* 52–56.

Phillips, N. B., Fuchs, L. S., Fuchs, D., & Hamlett, C. L. (1996). Instructional variables affecting student achievement: Case studies of two contrasting teachers. *Learning Disabilities Research & Practice, 11,* 24–33.

Porter, A. (1989). A curriculum out of balance: The case of elementary school mathematics. *Educational Researcher, 18*(5), 9–15.

Schunk, D. H. (1985). Participation in goal setting: Effects on self efficiacy and skills of learning disabled students. *Journal of Special Education, 19*, 307–317.

Shiah, R., Mastropieri, M. A., Scruggs, T. E., & Fulk, B. J. M. (1995). The effects of computer-assisted instruction on the mathematical problem solving of students with learning disabilities. *Exceptionality, 5,* 131–161.

Smith, D. D., & Lovitt, T. C. (1976). The differential effects of reinforcement contingencies on arithmetic performance. *Journal of Learning Disabilities, 9,* 32–40.

Stecker, P. M., & Fuchs, L. S. (2000). Effecting superior achievement using curriculum-based measurement: The importance of individual progress monitoring. *Learning Disabilities Research & Practice, 15*, 128–134.

Stedman, L. C. (1997). International achievement differences: An assessment of a new perspective. *Educational Researcher, 26,* 4–15.

Top, B. I., & Osguthorpe, R. T. (1987). Reverse-role tutoring: The effects of handicapped students tutoring regular class students. *Elementary School Journal, 87,* 413–424.

Trifiletti, J. J., Frith, G. H., & Armstrong, S. (1984). Microcomputers versus resource rooms for LD students: A preliminary investigation of the effects on math skills. *Learning Disabilities Quarterly, 7,* 69–76.

Walberg, H. J. (1984). Improving the productivity of America's schools. *Educational Leadership, 41*(8), 19–27.

Webb, N. M., Troper, J., & Fall, J. R. (1995). Constructive activity and learning in collaborative small groups. *Journal of Educational Psychology, 87,* 406–423.

Weigman, D. A., Dansereau, D. F., & Patterson, M. E. (1992). Cooperative learning: Effects of role playing and ability on performance. *Journal of Experimental Education, 60,* 109–116.

Woodward, J., Baxter, J., & Robinson, R. (1999). Rules and reasons: Decimal instruction for academically low achieving students. *Learning Disabilities Research & Practice, 14,* 15–24.

Woodward, J., Howard, L., & Battle, R. (1997). Learning subtraction in the third and fourth grade: What develops over time. Tech. Rep. No. 97-21. Tacoma, WA: University of Puget Sound.

Xin, Y. P., & Jitendra, A. K. (1999). The effects of instruction in solving mathematical word problems for students with learning problems: A meta-analysis. *Journal of Special Education, 32,* 207–225.

Zook, K. B., & DiVesta, F. J. (1989). Effects of overt controlled verbalization and goal-specific search on acquisition of procedural knowledge in problem solving. *Journal of Educational Psychology, 81,* 220–225.